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Kenneth H. Rosen



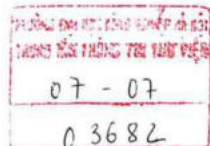
Discrete Mathematics and Its Applications

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Eighth Edition

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Kenneth H. Rosen

formerly AT&T Laboratories





DISCRETE MATHEMATICS AND ITS APPLICATIONS

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About the Author

Kenneth H. Rosen received his B.S. in Mathematics from the University of Michigan, Ann Arbor (1972), and his Ph.D. in Mathematics from the University of Michigan, Ann Arbor (1975). He completed his thesis in number theory under the direction of Professor G. B. Segre in 1975. In 1976, he joined the faculty of the University of Michigan, Ann Arbor. In 1982, he held positions at the University of California, San Diego; the University of Columbus; and the University of Maine. He has published numerous papers in mathematics. He enjoyed a long career as a mathematician at Bell Laboratories (and AT&T Laboratories). After leaving Bell Labs, he taught at Monmouth University. He has published papers on coding theory, and data security. After leaving Monmouth University, he joined the faculty of the University of New Hampshire, where he continues his research in computer security and cryptography, and in the theory of computation.

Dr. Rosen has published numerous papers in mathematics, and books on mathematical modeling. He is the author of *Discrete Mathematics and Its Applications*, published by Pearson, which has been translated into Chinese. He is also the author of *Discrete Mathematics*, McGraw-Hill, currently in its eighth edition. His books have sold more than 450,000 copies in North America and more than 150,000 copies throughout the rest of the world. His books have been translated into Spanish, French, Portuguese, and German. He is also the author of *UNIX: The Complete Reference*, published by Osborne/McGraw-Hill, and *UNIX Tips Ever*, all published by Osborne/McGraw-Hill. He has also published 150,000 copies, with translations into Chinese.

Preface

In writing this book, I was guided by the desire to present discrete mathematics in a clear and accessible manner, with the concepts and results rigorously demonstrated. My goal was to show students, who are often skeptical, the mathematical foundations they need. I wanted to give students an understanding of important concepts and how these concepts are important for applications. I wanted to achieve these goals without watering down the content.

For the instructor, my purpose was to provide a collection of proven pedagogical techniques in a format that is easy to use. I included a variety of materials that they could use to tailor the book to their most appropriate manner for their own teaching style and goals.

I have been extremely gratified by the response to this book by more than one million students around the world. It has been translated into many different languages. The success of the book is due in part to the feedback and suggestions of the more than 600 North American instructors who have used it. In the world, where this book has been successful, the appeal and effectiveness of this book have been demonstrated, and the significant investments that

and the significant investments that have been made in the field. This text is designed for a one-semester course by students in a wide variety of majors. College algebra is the only prerequisite. Some maturity is needed to study discrete mathematics. The text is designed to meet the needs of almost any student, is highly flexible and extremely complete. It is a textbook, but also to serve as a valuable reference in and professional life.

Goals of a Discrete Mathematics Course

A discrete mathematics course has a specific set of mathematical facts and how to use them. It teaches students how to think logically and how to use mathematical reasoning and the discrete mathematics interwoven in this text: mathematical logic, algorithmic thinking, and applications. A good course should carefully blend and balance these three aspects.

1. *Mathematical Reasoning:* Students should be able to comprehend, and construct mathematical arguments, mathematical logic, which serve as the foundation of proof. Both the science and the art of proof.